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Hypothetical Learning Trajectory in Place Value Concepts in Elementary School

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Abstrak

Teori Piaget telah banyak berpengaruh terhadap desain pembelajaran. Pembelajaran yang berorientasi pada guru (*teacher centere)* berubah menjadi berorientasi pada siswa *(student centere)*. Hal ini berarti bahwa faktor siswa menjadi hal yang utama dan harus diperhatikan dalam membuat suatu desain pembelajaran. Perumusan *Hypothetical Learning Trajectory* sebagai pedoman pelaksanaan pembelajaran sekaligus sebagai suatu tindakan antisipatif terhadap kemungkinan masalah yang dihadapi siswa dalam proses pembelajaran. Artikel ini menyajikan contoh perumusan *Hypothetical Learning Trajectory* untuk pembelajaran nilai tempat di kelas 1 sekolah dasar.
Kata Kunci: *Hypothetical Learning Trajectory*, Nilai Tempat.

Abstract

Piaget's theory has had a lot of influence on learning design. Teacher-oriented learning (teacher center) changed to student-oriented (student center). This means that the student factor is the main thing and must be considered in making a learning design. Formulation of Hypothetical Learning Trajectory as a guideline for implementing learning as well as an anticipatory action against possible problems faced by students in the learning process. This article presents an example of the formulation of a Hypothetical Learning Trajectory for place-value learning in grade 1 elementary school.
Keywords: Hypothetical Learning Trajectory, Place Value.

I. INTRODUCTION

Mathematics is a field of study that is taught from elementary to university level. Mathematics subjects at every level of formal education are seen as having a very important role in the formation of human beings who have the ability to think in the process of solving problems in everyday life. One of the objectives of learning mathematics contained in the elementary school curriculum is to develop computing skills in everyday life. During the computational process, the concept of place value is an important concept in the number system used in the sequence or position of a number among other numbers.

In general, learning the concept of place value is carried out directly at the formal stage following the pattern in the textbook. So that the concept of place value can function to direct students to able to understand the concept of the value of a number in order to anticipate an error in reading and writing multidigit numbers. However, in reality, it shows that students' understanding of place value material is still very weak.

In accordance with Hartono's statement (2013: 142), students often make mistakes in writing number symbols and number names, mistakes occur when students determine place value and number value, and mistakes in writing number symbols based on place value. This error occurs because students have difficulty understanding the concept of place value. As a result, students' high-level cognitive abilities are very weak because the usual learning activities only encourage students to think at a lower level (Herman, 2006; Fitri, Fitri, & Jufri, 2022).

If the teacher's way of teaching is less varied and tends to follow what is in the textbook as above, then learning becomes less meaningful or even meaningless for students, so students do not understand the concept of place value taught by the teacher. Learning activities that are only communicated by the teacher to students in one direction such as pouring water into a glass, students only memorize concepts and are less able to use these concepts if they encounter problems in real life related to the concepts they have (Zaneta, 2022).

This can be seen from the results of observations and interviews regarding the results of exercises carried out by students when students are directed to distinguish the writing of place values and numeric values, it is likely that they will experience errors. For example, an error when writing the number "twenty-two" is written with 20 and 2 (202). Another example is the operation to calculate the place value of 22 x 44 as shown in Figures 1 and 2 below:

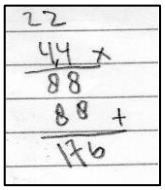

Figure 1. Student Work Results in 1

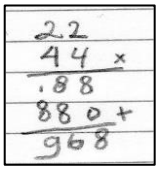

Figure 2. Student Work Results in 2

Based on the picture above, procedures that involve base ten number structures such as storing in place values that involve

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addition concepts with storing techniques also tend to be errors.

In addition, based on the results of a field study by researchers at an elementary school in Garut Regency, it appears that students do not understand and interpret the place value of a number when students are shown the symbol for the number "three hundred and three" (303), then students are asked to state the position from number 3 in that number, there are several different answers given by students. First, there are students who are able to answer correctly that the position of the number 3 on the far right is units and the number 3 on the far left is hundreds. Second, there were students who answered that the number 3 on the far left was units and the number 3 on the right was tens. Third, there were even students who answered that the number 3 on the right was units and the number 3 on the left was tens. Based on the second and third alternative answers, it shows that students still do not understand the place value material, even though the material is an important foundation in developing numeracy skills (addition, subtraction, place value, and division).

Rosnawati also revealed that the low achievement of students who answered these questions was due to a poor understanding of place value. An erroneous understanding of place value will result in errors in arithmetic operations, especially addition. Of course, this situation becomes complicated because it will affect the next mathematical concept.

As a solution to this problem, this research deems it necessary to develop strategies or ways for students to approach learning situations and their reasoning is very varied. Donovan and Bransford (2005) in their book How Students Learn History, Mathematics, and Science In The Classroom describe that there are various paths or thinking processes of different children in understanding mathematics. Children clearly vary greatly in their mental development. It is possible to introduce them to various representations that will help develop their strengths and accommodate their limitations. Each student has achieved their level of achievement through different routes in building the skills they have, the attitudes they inculcate, and the modes of thinking they rely on (Wahyudin, 2010; Nuraida, 2018).

Sarama and Clements (2009) emphasized "Understanding the level of thinking of the class and individuals in that class is key in serving the needs of all children". Understanding children's thinking levels in the classroom is key to serving the needs of all children. Effective learning requires teachers to meet students' needs and help build knowledge that students know. So, teachers should understand learning trajectories, how children think and learn mathematics, and how to help children learn better.

The term learning trajectory (learning flow) was first used by Simon (1995; Afriansyah, 2022) where there are three main components of the learning trajectory, namely: learning goals (learning goals), learning activities (learning activities), and hypothetical learning processes (hypothetical learning process).

Learning objectives as the first component indicate the need to formulate learning objectives as a form of results that we will aim for or achieve after the learning process. Determining learning objectives is very useful in determining the direction and learning strategies to be used. Based on the learning objectives that have been formulated, learning activities (learning activities) as a "way" to achieve learning objectives can be designed. Learning activities are organized into several sub-activities with sub-sub-objectives of learning. The last component is the student learning process hypothesis which is useful for designing alternative actions or strategies to overcome various problems that students may face in the learning process.

Learning Trajectories or how children think when they learn to achieve specific goals in mathematical concepts, through a series of instructional tasks designed to elicit mental processes or actions that are hypothesized to move children through the development of children's thinking development (Afriansyah & Turmudi, 2022).

There are tips on how to build Learning Trajectories expressed by Stephens and Armanto (2010) in their writing How to Build Powerful Learning Trajectories for Relational Thinking in the Primary School Years. This paper aims to describe an idea of a useful learning trajectory to see how the achievement of learning numbers and algebra in the lower grades. The results obtained are in the form of a learning trajectory that is designed in such a way.

II. METHOD

This research uses a type of design research that designs a Hypothetical Learning Trajectory or student learning trajectory on place value material in elementary schools using the guided discovery method. The design research model used is the Gravemeijer and Cobb (2006) model which defines design research into three stages, namely preparing for the experiment, designing the experiment, and retrospective analysis.

The steps are explained as follows:

A. Preparing for the Experiment

At this stage, the activity starts with the formation of the 1st Focus Group Discussion (FGD) in a sequence starting from formulating an Empirical Learning Trajectory pattern for whole number place values; analyzing learning objectives; determining the initial conditions of research; discussing the initial conjecture of place value HLT to be developed; determine the characteristics of the class and the role of the teacher; and determine the theoretical goals to be achieved.

B. Design Experiment

At this stage, it begins with the implementation of the experimental design that has been prepared as well as testing and developing place value HLT.

C. Retrospective Analysis

Through this stage, the activity begins with the formation of the Second FGD, namely to carry out activities to analyze the data that has been obtained to find out whether it supports and is in accordance with the conjecture that has been designed.

Based on the steps above, this research activity was designed using the agreed

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modules, designed lesson plans, and observation sheets, which the researchers and participants would work together in implementing research activities in learning.

The instrument as the main measuring tool in this study is the test instrument, interviews, and observations.

III. RESULT AND DISCUSSION

Piaget's theory has had a lot of influence on learning design. Teacher-oriented learning turns into student-oriented. This means that the student factor is the main thing and must be considered in making a learning design. The ideal learning process cannot be separated from the learning planning and design process. Learning Implementation Plans or lesson plans are a concrete form of the planning and design process of learning. However, in reality, a Learning Implementation Plan only contains things that are formal in nature in the form of a "standard package" of learning, namely a brief description of opening activities, core activities, and closing activities. It is very rare for teachers to prepare alternative hypotheses for problem-solving strategies used by students.

The existence of an alternative hypothesis of strategies used by students will assist teachers in determining strategies for dealing with possible difficulties faced by students. The importance of a hypothetical learning trajectory can be analogous to planning a travel route. If we understand the possible routes to our destination then we can choose a good route. In addition, we can also solve the problems we face on the way if we understand the route. For example, we can anticipate running out of fuel if we know the position of the gas station. Meanwhile, model development is very important to bring students' informal knowledge (student initial capital formed through experience-based activities) towards formal mathematical concepts (as the ultimate goal of learning mathematics).

As Simon (1995) has stated, there are three main components in the learning trajectory, namely learning goals, learning activities, and hypothetical learning processes.

A. Learning Objectives

When referring to the curriculum, place value is the competence for number operations for first-grade students in semester 1. In accordance with KIKD Mathematics Class 1, namely "3.2. Explaining numbers up to two digits and the area values that make up the number symbols using a collection of concrete objects and how to read them" (Revised edition of the 2013 Curriculum)

B. Learning Activities

Based on the learning objectives that have been formulated, learning activities can be designed. However, the thing that must be done before designing learning activities is to understand the unity of the place value concept as a whole so that the sequence or stages of learning activities are in accordance with the basic concept of place value.

C. Student learning process hypothesis

One very important element of the Hypothetical Learning Trajectory is the hypothesis of the student's learning

process. When designing learning, teachers should develop hypotheses (conjectures) or student reactions at each learning stage. In the early stages of learning planning, the hypothesis is based on pre-knowledge that students already have and based on experience or practice of previous place value learning. (Gravemeijer, 2004).

This study uses worksheets, clinical interviews, observation, and documentation.

1) Description of Observation Results

An example of a student learning hypothesis in learning about place value through observation activities can be obtained in that at the first meeting the students were very enthusiastic and very interested in the learning provided by the researcher because they saw tools or media that they had never seen in their school before.

At meeting 2 every aspect had a good effect on students, the response and enthusiasm of students in learning were fairly good. In the second discovery, students can listen and understand the teacher's explanation even though students are not so confident when speaking in front of their friends and asking the researcher. Furthermore, at meeting 3 each aspect had a good effect on students, the response and enthusiasm of students in learning were quite good in the third finding because students could listen and understand the teacher's explanation well.

In addition, the learning alternatives used are varied, including:

a) Alternative Learning 1

From the results of the process of implementing implementation of HTL learning regarding place value material 1-10, the researcher first gave apperception to students so that they were able to absorb the knowledge conveyed by the researcher. The researcher reminds students about the place value material beforehand by providing examples of learning place value using concrete objects, after reviewing the researcher applies HTL learning theory in the concept of place value learning. To facilitate the application of learning the researcher has prepared media or so-called learning aids, Student Worksheets (LKS) so that students really understand the concepts being taught by researchers, bearing in mind that in accordance with the development of grade 1 elementary school students, they are at the concrete operational stage, for more facilitate the application of learning theory, namely by using concrete objects, namely by transferring images as shown below:

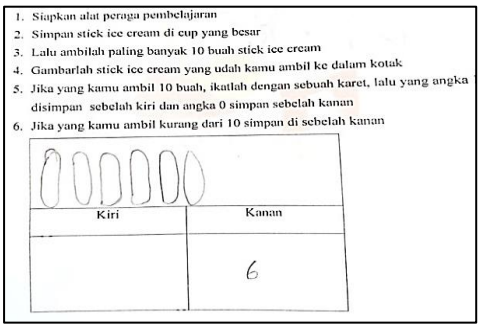

Figure 3. Results of Work on Alternative 1

b) Alternative Learning 2

In the process of implementing the application of HTL learning regarding place value material 10-15, before applying the theory the researcher first gives an apperception to students from the previous meeting so that they

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are able to absorb the knowledge conveyed by the researcher. The researcher reminds students about the place value material beforehand by giving examples of learning place value that has been done in the first meeting using concrete objects, after reviewing the researcher applies HTL learning in the concept of place value learning. To facilitate the application of HTL the researcher has prepared media or so-called learning aids, student worksheets (LKS) so that students really understand the concepts being taught by researchers, bearing in mind that in accordance with the development of grade 1 elementary school students, they are at the concrete operational stage, to further facilitate the application of learning theory by using concrete objects. The media used in meeting 2 used ball props, and a magic bag. The three students were quite enthusiastic and curious when they saw the ball.

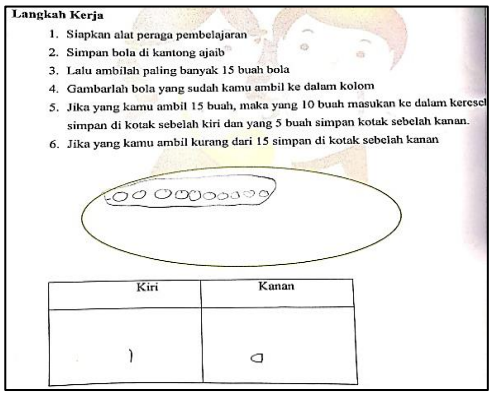

Figure 4. Results of Work on Alternative 2

c) Alternative Learning 3

The results of the process of implementing the implementation of HTL learning regarding place value material 15-20, before applying the theory the researcher first gave an apperception to students from the previous meeting so that they were able to absorb the knowledge conveyed by the researcher. The researcher reminds students about the place value material beforehand by giving examples of learning place value which has been reviewed in the second meeting using concrete objects, after reviewing the researcher applies HTL theory in the concept of learning place value. To facilitate the application of HTL, researchers have prepared media or so-called learning aids, student worksheets (LKS) so that students really understand the concepts being taught by researchers, bearing in mind that according to the development of grade 1 elementary school students, they are at the concrete operational stage, to further facilitate the application of learning theory by using concrete objects.

The media used in meeting 3 used Lego props. The three students were quite enthusiastic and curious when they saw Lego.

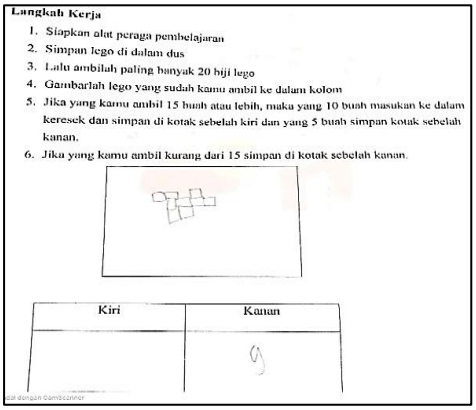

Figure 5. Results of Work on Alternative 3

2) Description of the Clinical Interview

From the results of interviewing participants during the teaching and learning process, it was found that each participant had different characteristics when participants were asked to take the media in front of them, each participant took a lot or a little and they had reasons when they took a lot or a little. the reason is that they like the number or if the number has meaning for them, knowing whether the media used is appropriate or in accordance with the participant's characteristics. Of course, it was different, some were appropriate and some were not because each participant had their own preferences, at meeting 1 they used ice cream stick props so the participants were quite enthusiastic because before when studying in class they had never used props they were only focused on the inside. government-supplied books.

Furthermore, the results of the participant interviews at the 2nd meeting when the KBM process took place, it was found that each participant had different characteristics, the same as the previous meeting, when participants were asked to take the media in front of them, each participant took a lot or a little and they had the reason when they take a lot or a little, the reason is because they like the number or also the number has meaning for them, knowing whether the media used is right or in accordance with the participant's characteristics. Of course, it was different, some were appropriate and some were not because each participant had their own preferences, at this 2nd meeting using ball props the participants were quite enthusiastic even though the ball was often seen in their surroundings.

Finally, the results of the participant interviews at the last meeting when the KBM process took place, found that each participant had different characteristics as in the previous meeting, when participants were asked to take the media in front of them, each participant took a lot or a little and they had reasons when they take a lot or a little, the reason is because they like the number or the number has meaning for them, knowing whether the media used is right or in accordance with the participant's characteristics. Of course, there are differences, some are suitable and some are not because each participant has their own preferences, in this last meeting, Lego props were used because Lego is a toy that students rarely encounter besides being expensive, children rarely have it so students are enthusiastic. extraordinary.

Based on this, not only understanding the mathematical concepts, but also knowing their application in everyday life and creating a conducive and enjoyable learning atmosphere to direct participants to achieve the goals of learning mathematics optimally.

Next, a description to see how successful the implementation of HLT is in the concept of place value, the researcher describes the discussion of the research data into 3 points, namely: alternative learning 1, alternative learning 2, and alternative learning 3.

Based on the results of the processing and analysis of the data above, it is found that the process of applying HLT in the learning concept of place values for tens to tens is very well applied to students in

grade 1 elementary school, especially when accompanied by ball and lego learning media that makes it interesting for students and makes it easier to explain value material. the place. Furthermore, at the 2nd meeting the students had started to understand what they had to do, unlike the first meeting, they were just silent and focused on looking at the props without listening to the teacher's explanation, but in writing the symbol for the number of place values, there were still students who were confused about placing the value of the results they were working on. . Therefore, it is necessary to use teaching aids to support longer retention or longer memory, easy to remember, besides that discovery learning has learning outcomes that have a better transfer effect than other learning outcomes. Besides that, the presence of teaching aids (media) can attract students' learning interest which of course will affect student learning achievement at school (Puspitarini & Hanif, 2019).

IV. CONCLUSION

Based on a brief description of the hypothetical learning trajectory, several conclusions can be drawn as follows: 1) Hypothetical Learning Trajectory can provide teachers with an understanding of how important it is to pay attention to students' initial knowledge and also differences in students' abilities in preparing learning designs; 2) Hypothetical Learning Trajectory can be used as a teacher's guide in dividing the stages of learning, namely by making several sub-objectives of learning to achieve the main learning objectives; and 3) Hypothetical Learning Trajectory is useful as a guide for implementing learning as well as providing various alternative strategies or scaffolding to help students overcome difficulties in understanding the concepts being studied.

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